EXERCISE – IV ADVANCED SUBJECTIVE QUESTIONS

1. Find the point of intersection of the tangents drawn to the curve $x^2y = 1 - y$ at the points where it is intersected by the curve $xy = 1 - y$.

2. Find all the lines that pass through the point (1, 1) and are tangent to the curve represented parametrically as $x = 2t - t^2$ and $y = t + t^2$.

3. A function is defined parametrically by the equations

$$f(t) = x = \begin{cases} 2t + t^2 \sin\frac{1}{t} & \text{if } t \neq 0 \\ 0 & \text{if } t = 0 \end{cases} \text{ and}$$

$$g(t) = y = \begin{cases} \frac{1}{t}\sin t^2 & \text{if } t \neq 0 \\ 0 & \text{if } t = 0 \end{cases}$$

Find the equation of the tangent and normal at the point for t = 0 if exist.

4. Find all the tangents to the curve $y = \cos(x + y)$, $-2\pi \le x \le 2\pi$, that are parallel to the line $x + 2y = 0$.

5. Show that the normals to the curve $x = a(\cos t + t \sin t)$; $y = a(\sin t - t \cos t)$ are tangent lines to the circle $x^2 + y^2 = a^2$.

6. If the tangent at the point (x_1, y_1) to the curve $x^3 + y^3 = a^3$ meets the curve again in (x_2, y_2) then show that $\frac{x_2}{x_1} + \frac{y_2}{y_1} = -1$.

7. The tangent at a variable point P of the curve $y = x^2 - x^3$ meets it again at Q . Show that the locus of the middle point of PQ is $y = 1 - 9x + 28x^2 - 28x^3$.

8. Show that the condition that the curves $x^{2/3} + y^{2/3} = c^{2/3}$ & $(x^2/a^2) + (y^2/b^2) = 1$ may touch if $c = a + b$.

9. A curve is given by the equations $x = at^2$ & $y = at^3$. A variable pair of perpendicular lines through the origin 'O' meet the curve at P & Q . Show that the locus of the point of intersection of the tangents at P & Q is $4y^2 = 3ax - a^2$.

10. A and B are points of the parabola $y = x^2$. The tangents at A and B meet at C. The median of the triangle ABC from C has length 'm' units. Find the area of the triangle in terms of 'm'.

11. (a) Find the value of n so that the subnormal at any point on the curve $xy^n = a^{n+1}$ may be constant.
(b) Show that in the curve $y = a.\ell n(x^2 - a^2)$, sum of the length of tangent & subtangent varies as the product of the coordinates of the point of contact.

12. If the two curves $C_1 : x = y^2$ and $C_2 : xy = k$ cut at right angles find the value of k.

13. A man 1.5 m tall walks away from a lamp post 4.5 m high at the rate of 4 km/hr.
(i) how fast is the farther end of the shadow moving on the pavement ?
(ii) how fast is his shadow lengthening ?

14. A water tank has the shape of a right circular cone with its vertex down . Its altitude is 10 cm and the radius of the base is 15 cm . Water leaks out of the bottom at a constant rate of 1 cm^3/sec . Water is poured into the tank at a constant rate of C cm^3/sec. Compute C so that the water level will be rising at the rate of 4 cm^3/sec at the instant when the water is 2 cm deep.

15. Sand is pouring from a pipe at the rate of 12 cm^3/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always 1/6th of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm.

16. An open can of oil is accidently dropped into a lake; assume the oil spreads over the surface as a circular disc of uniform thickness whose radius increases steadily at the rate of 10 cm/sec. At the moment when the radius is 1 meter, the thickness of the oil slick is decreasing at the rate of 4 mm/sec, how fast is it decreasing when the radius is 2 meters.

17. A variable Δ ABC in the xy plane has its orthocentre at vertex 'B' , a fixed vertex 'A' at the origin and the third vertex 'C' restricted to lie on the parabola $y = 1 + \frac{7x^2}{36}$. The point B starts at the point (0, 1) at time $t = 0$ and moves upward along the y axis at a constant velocity of 2 cm/sec . How fast is the area of the triangle increasing when $t = \frac{7}{2}$ sec.

18. A circular ink blot grows at the rate of 2 cm^2 per second . Find the rate at which the radius is increasing after $2\frac{6}{11}$ seconds . (Use $\pi = \frac{22}{7}$)

19. Water is flowing out at the rate of 6 m^3/min from a reservoir shaped like a hemispherical bowl of radius R = 13 m. The volume of water in the hemispherical bowl is given by $V = \frac{\pi}{3} \cdot y^2 (3R - y)$ when the water is y meter deep. Find

(a) At what rate is the water level changing when the water is 8 m deep.

(b) At what rate is the radius of the water surface changing when the water is 8 m deep.

20. At time $t > 0$, the volume of sphere is increasing at a rate proportional to the reciprocal of its radius. At $t = 0$, the radius of the sphere is 1 unit and at $t = 15$ the radius is 2 units.

(a) Find the radius of the sphere as a function of time t.

(b) At what time t will the volume of the sphere be 27 times its volume at $t = 0$.